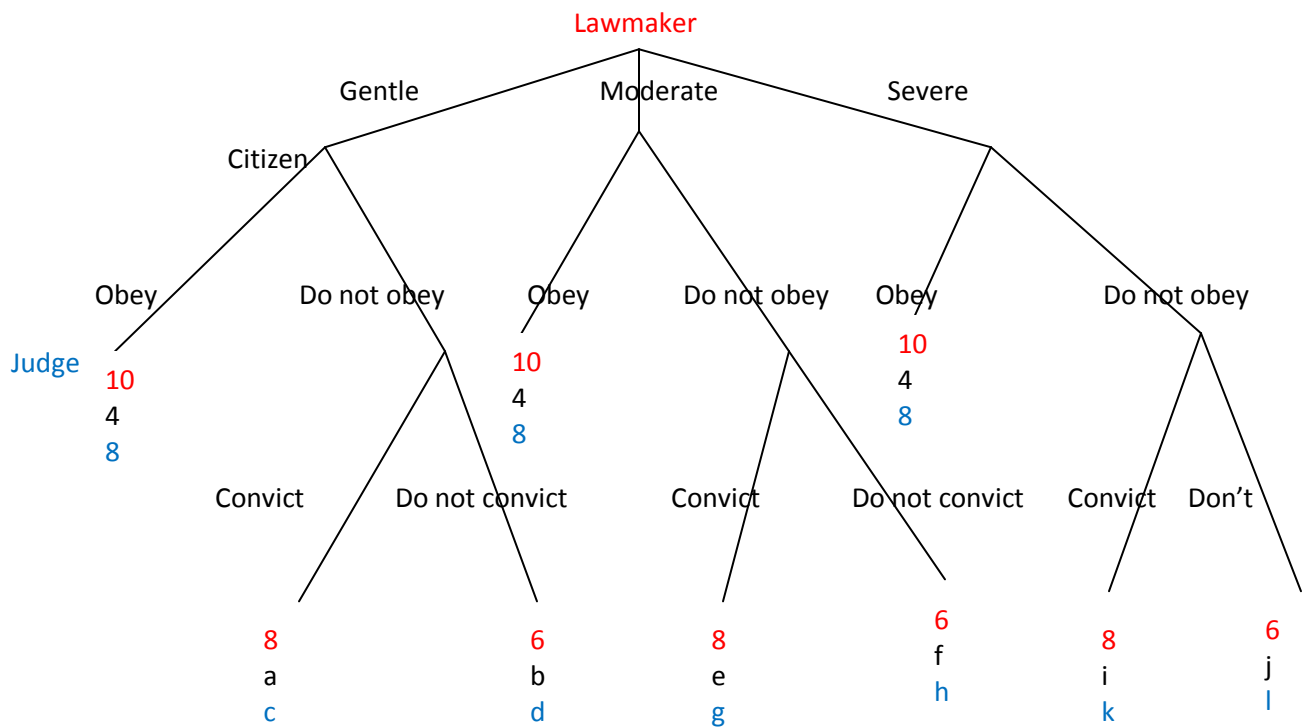(Please write legibly; if in doubt, type your assignment.)

This homework assignment is based on the material in chapters 7 and 8 of the course text.

1.   (Question 8, chapter 7) It is the closing seconds of a football game, and the losing team has just scored a touchdown. Now down by only one point, the team decides to go for a two-point conversion that, if successful, will win the game. The offense chooses between three possible running plays: run wide left, run wide right, and run up the middle. The defense decides between defending against a wide run and a run up the middle. The payoff to the defense is the probability that the offense does not score, and the payoff to the offense is the probability that it does score. Find all **pure** and **mixed**-strategy Nash equilibria. *This question is not easy; think carefully.*

<table>
<tr><td></td><td></td><td colspan="3" align="center">Offense</td></tr>
<tr><td rowspan="3">Defense</td><td></td><td>Run wide left</td><td>Run up middle</td><td>Run wide right</td></tr>
<tr><td>Defend against wide run</td><td>.6,.4</td><td>.4,.6</td><td>.6,.4</td></tr>
<tr><td>Defend against run up the middle</td><td>.3,.7</td><td>.5,.5</td><td>.3,.7</td></tr>
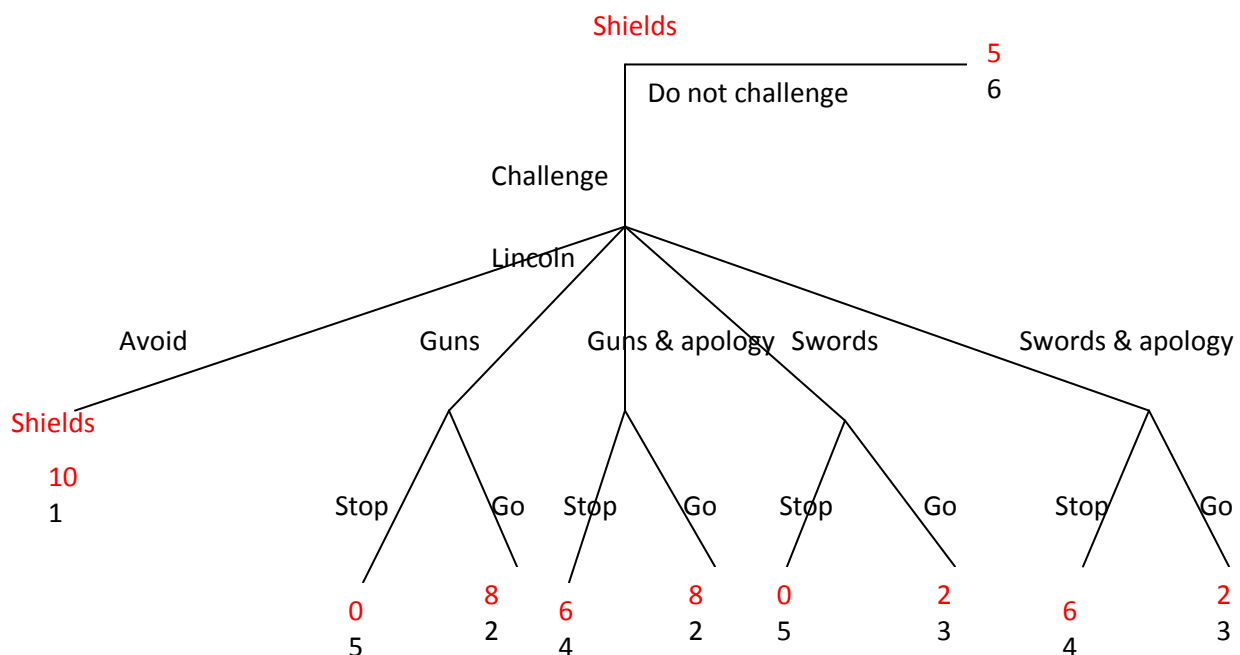</table>

2. (Question 4, chapter 8)   Benjamin Franklin once said, "Laws too gentle are seldom obeyed; too severe, seldom executed."   To flush out what he had in mind, the game [below] has three players: a lawmaker, a (typical) citizen, and a judge. The lawmaker chooses among a law with a gentle penalty, one with a moderate penalty, and a law with a severe penalty. In response to the law, the citizen decides whether or not to obey it. If she does not obey it, then the judge decides whether to convict and punish the citizen. Using subgame perfect Nash equilibrium, find values for the unspecified payoffs (those with letters, not numbers) that substantiate Franklin's claim by resulting in lawmaker's choosing a law with a moderate penalty.

Lawmaker

- Gentle
  - Citizen
    - Obey → Judge: 10, 4, 8
    - Do not obey
      - Convict → 8, a, c
      - Do not convict → 6, b, d
- Moderate
  - Obey → 10, 4, 8
  - Do not obey
    - Convict → 8, e, g
    - Do not convict → 6, f, h
- Severe
  - Obey → 10, 4, 8
  - Do not obey
    - Convict → 8, i, k
    - Don't → 6, j, l

The objective is to find values so that a subgame perfect Nash equilibrium has the following properties:
1) When the law is gentle, the citizen disobeys the law even though he will be convicted
2) When the law is moderate, the citizen obeys the law because he will be convicted
3) When the law is severe, the citizen disobeys the law because he will not be convicted.

3. (Question 6, chapter 8) In 1842, the Sangamo Journal of Springfield, Illinois, published letters that criticized James Shields, the auditor of the State of Illinois. Although the letters were signed "Rebecca," Shields suspected that it was state legislator Abraham Lincoln who penned the letters. As shown in [the figure below], Shields considered challenging Lincoln to a dual, and, as history records, Shields did challenge Lincoln. In response to a challenge, Lincoln could avoid the duel, or; if he chose to meet Shield's challenge, he had the right to choose the weapons. We will also allow Lincoln to decide whether to offer an apology of sorts. If he decides to go forward with a duel, then Lincoln has four choices: propose guns, propose guns and offer an apology, propose swords, or propose swords and offer an apology. In response to any of the four choices, Shields must decide to either go forward with the duel or stop the duel (and accept the apology if offered). Find all subgame perfect Nash equilibria.

4. Suppose you are analyzing a simple two-stage bargaining game, in which the first move is a proposal of how to divide $4, which must be either $3 for the proposer and $1 for the responder or $2 for each. The responder sees the proposal and either accepts (which implements the split) or rejects the offer, which results in $0 for each. The responder's strategy must specify a reaction to each possible proposal. First, indicate the pure strategy Nash equilibrium(ia). Then indicate how the concept of subgame perfect Nash equilibrium can eliminate seeming irrationality within a Nash equilibrium. Your answer should explain the value of the concept with respect to the usefulness of game theory and Nash equilibrium in general.

5. Carefully write the first section of your paper. This should include the introduction to your paper generally, the scenario you are modeling, the players, and their objectives. It also should include your extensive form diagram and your strategic form diagram. This part of your homework assignment should be typed (your extensive and strategic form diagrams may be handwritten and submitted with the problems) and sent to me via e-mail as a word document.